ANALYSIS OF FACTORIAL EXPERIMENTS BY BREAKING INTO GROUPS

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(Received in July, 1969)

INTRODUCTION

When the number of factors in an experiment of the type s^n is large, it becomes inconvenient first to write all the treatment combinations and then carry out operations on them as required through the methods due to Yates (1937), Box et al. (1954) and Good, I. J. (1958) for obtaining contrasts corresponding to various main effects and interaction components. We have discussed below a convenient method for their analysis through which the totality of treatment combinations can first be made into groups of equal size and then each group is analysed separately. Subsequently these results are analysed again for obtaining the final results.

METHOD

Let $A_1, A_2, \ldots\ldots, A_n$ be n factors each at s levels. Instead of writing the whole of the s^n treatment combinations along with their observation totals as required for applying the methods, we make them into equal groups of size s^n $(m<n)$ each so that there will be s^{n-m} groups. These groups may conveniently be made (without any regard to blocking adopted at the time of construction of the design), say, corresponding to the confounding of all the main effects and interactions of the last $(n-m)$ factors, viz., $A_{m+1}, A_{m+2}, \ldots\ldots, A_n$. The first group is then written as required in Yates' technique or its extension using the first m factors $A_1, A_2, \ldots\ldots A_m$. The ith of the other groups is then obtained by multiplying each of the treatment combinations in the first group by S_i and writing them in the corresponding order where S_i is the ith treatment combination of the last $(n-m)$ factors when written in the order required by Yates'

method for their analysis. The first group is now taken and the observation totals are written against the corresponding treatment combinations. The usual operations are carried out on these observation totals, there being m cycles of operations as required for s^m treatment combinations. Each of the other groups, say, the ith group is also analysed as the first group by suppressing (S_i) in each treatment combination. Next let us consider the s^{n-m} contrasts available against any treatment combination, say, t_j in the first group and all its multiple by S_i, for different values of i. These s^{n-m} contrasts are then written in a column following the order of S_i and without suppressing S_i. With these s^{n-m} contrasts $(n—m)$ cycles of operations are carried out as required by the method. From this table we shall get the s^{n-m} contrasts appropriate for the s^n design corresponding to interactions $t_j a_i$ where a_i varies over the treatment combinations of the last $(n—m)$ factors only. By varying t_j over the treatments in the first group, we shall get all the s^n contrasts including the grand total.

Though the method has been discussed for the s^n experiment, it can be extended in a straightforward way to the case of asymmetrical designs.

In order to clarify the technique further we have illustrated it by analysing (1) a symmetrical and (2) an asymmetrical factorial experiments.

Case (1) Analysis of 2^5 factorial experiment.

Let the factors be donoted by A, B, C, D and E. We make four groups each of size 8 corresponding to the confounding of the main effects and interaction of D and E. The groups, together with a fictitious set of data, are shown below with their analysis.

Analysis of Group I

Treat. Comb.	*Obs. total*	*Cycles 1*	*2*	*3*
1	1	3	13	41
a	2	10	28	1
b	4	16	3	3
ab	6	12	—2	—5
c	7	1	7	15
ac	9	2	—4	—5
bc	8	2	1	—11
abc	4	—4	—6	—7

*Analysis of Group II**

Treat. Comb.	*Obs. total*	*Cycles 1*	*2*	*3*
d	3	5	8	34
ad	2	3	26	0
bd	1	15	0	—6
abd	2	11	0	0
cd	7	—1	—2	18
acd	8	1	—4	0
bcd	6	1	2	—2
abcd	5	—1	—2	—4

*Group II combinations are obtained by muitiplying the group I combinations by d and writing the product in the same order.

Analysis of Group III
(Obtained from group I by multiplying by e)

Treat. Comb.	*Obs. total*	*cycles 1*	*2*	*3*
e	3	7	9	23
ae	4	2	14	3
be	1	5	1	−1
abe	1	9	2	−1
ce	2	1	−5	5
ace	3	0	4	1
bce	4	1	−1	9
abce	5	1	0	1

Analysis of Group IV
(Obtained from group I by multiplying by de)

Treat. Comb.	*Obs. total*	*cycles 1*	*2*	*3*
de	4	10	22	36
ade	6	12	14	−2
bde	7	9	0	−2
abde	5	5	−2	−4
cde	5	2	2	−8
acde	4	−2	−4	−2
bcde	3	−1	−4	−6
abcde	2	−1	0	4

Now we form 8 groups of size 4 and analyse them to get the final results.

**Treat. Comb.*	*Contrast values*	*Final contrasts 1*	*2*
1	41	75	134
d	34	59	6
e	23	−7	−16
de	36	13	20

b	3	−3	−6
bd	−6	−3	−10
be	−1	−9	0
bde	−2	−1	8

c	15	33	30
cd	18	−3	−10
ce	5	3	−36
cde	−8	−13	−16

bc	−11	−13	−10
bcd	−2	3	−6
bce	9	9	16
bcde	−6	−15	−24

Treat. Comb.	*Contrast values*	*Final contrasts 1*	*2*
a	1	1	2
ad	0	+1	−6
ae	3	−1	0
ade	−2	−5	−4

ab	−5	−5	−10
abd	0	−5	2
abe	−1	5	0
abde	−4	−3	−8

ac	−5	−5	−6
acd	0	−1	2
ace	1	5	4
acde	−2	−3	−8

abc	−7	−11	−6
abcd	−4	5	6
abce	+1	3	16
abcde	4	3	0

*These are the combinations entering as the first entry in the above 4 tables (first series of tables) appearing in the same order as the tables. The *i*th combination in the above tables (first series) constitute the *i*th table of the second type (second series of tables). The *i*th table combinations (second series) are also the products of the first table (second series) combinations with the *i*th treatment combination in the first table (first series).

The S.S. due to any interaction contrast, say, AB can now be obtained by squaring the contrast against ab, namely, -10 (as read from the second series of tables) and dividing it by $32r$. Thus S.S. due to $AB=(-10)^2/32r$, where r is the number of replications.

Case (2) Analysis of $2^2 \times 3^2$ factorial experiment

Let there be 2 factors, A and B each at 2 levels and 2 factors C and D each at 3 levels. Let us decide to make groups of size 2×3, so that there will be 6 groups. We make the groups corresponding to the confounding of the main effects and interactions of B and D. The data used in the example are again fictitious.

Analysis of Group I

Treat. Comb.	Obs. total	1	2
1	2	7	20
a	5	10	-2
c	7	3	4
ac	3	3	4
c^2	2	-4	-10
ac^2	1	-1	10

Analysis of Group II

Treat. Comb.	Obs. total	1	2
b	3	7	21
ab	4	6	9
bc	1	3	-1
abc	5	1	-3
bc^2	2	4	3
abc^2	6	4	-3

Analysis of Group III

d	1	3	18
ad	2	12	4
cd	4	3	0
acd	8	1	2
c^2d	2	4	-18
ac^2d	1	-1	-8

Analysis of Group IV

bd	6	9	18
abd	3	6	0
bcd	2	3	6
$abcd$	4	-3	-4
bc^2d	1	2	0
abc^2d	2	1	-6

Analysis of Group V

Treat. Comb.	Obs. total	1	2
d^2	1	10	21
ad^2	9	5	11
cd^2	2	6	4
acd^2	3	8	6
c^2d^2	2	1	6
ac^2d^2	4	2	8

Analysis of Group VI

Treat. Comb.	Obs. total	1	2
bd^2	8	13	28
abd^2	5	9	-2
bcd^2	3	6	7
$abcd^2$	6	-3	-1
bc^2d^2	4	3	1
abc^2d^2	2	-2	-11

Now we form 6 groups of size 6 and analyse them to get the final results.

Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2	Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2
1	20	41	126	*a*	−2	7	20
b	21	36	8	*ab*	9	4	−6
d	18	49	−8	*ad*	4	9	−2
bd	18	1	−6	*abd*	0	11	24
d^2	21	0	18	ad^2	11	−4	8
bd^2	28	7	8	abd^2	−2	−13	6

Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2	Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2
c	4	3	20	*ac*	4	1	4
bc	−1	6	4	*abc*	−3	−2	−20
cd	0	11	−8	*acd*	2	5	−4
bcd	6	−5	−8	*abcd*	−4	−7	0
cd^2	4	6	2	acd^2	6	−6	10
bcd^2	7	3	−14	$abcd^2$	−1	−7	−2

Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2	Treat. Comb.	Contrast value	Final contrast 1	Final contrast 2
c^2	−10	−7	−18	ac^2	1	−2	−19
bc^2	3	−18	26	abc^2	−3	−14	−21
c^2d	−18	7	−14	ac^2d	−8	−3	1
bc^2d	0	13	−18	abc^2d	−6	−4	15
c^2d^2	6	18	36	ac^2d^2	8	2	23
bc^2d^2	1	−5	−28	abc^2d^2	−11	−19	−27

The S.S. due to any interaction contrast, say, BC_Q can now be obtained by squaring the contrast against bc^2, namely, 26 (as read from second series of tables) and dividing it by 72 *r*, where *r* is the number of replications and C_Q stands for the quadratic effect of the factor *C*.

DISCUSSION

To sum up if there be s^n combinations, we first make s^{n_1} groups each of s^{n-n_1} combinations. Next each group is analysed separately, which we may call the first operation. Next the results of the first operation are analysed as indicated and this we may call the second operation. If we choose s^{n-n_1} as an easily manageable size, it may happen that s^{n_1} is too large and may thus be not very suitable for analysis through the second operation. In such situation the second operation analysis can be performed through similar grouping and such grouping may be repeated as far as required.

Chances of errors through the present method are likely to be smaller as only small operations are handled each time.

As in complete factorial experiments, the totality of treatment combinations in fractional factorials can also be broken into groups when the number of treatment combinations is large. Once the treatment combinations are written in a systematic order, they can be broken into groups of equal size on the basis of the existing factors and analysed as before.

SUMMARY

When a large number of factors is involved in a factorial experiment, the analysis through the methods due to Yates (1937), Box et al. (1954) and Good, I.J. (1958) become complicated as a very large number of combinations has to be taken and operated upon. We have given a modified method whereby the treatment combinations are first made into the suitable groups and then each group is analysed separately. The separate analyses of these groups are then combined to get the final results.

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